4/EH–29 (iv) (Syllabus–2015)

2017

(April)

MATHEMATICS

(Elective/Honours)

(Statics and Dynamics)

(GHS–41)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking one from each Unit.

Answer Statics and Dynamics in two separate books

UNIT—I

1. *(a)* State Lami's theorem. Forces P, Q, R acting along $\overrightarrow{IA}$, $\overrightarrow{IB}$, $\overrightarrow{IC}$, where I is the incentre of the triangle ABC, are in equilibrium. Show that

$$P:Q:R=\cos\frac{1}{2}A:\cos\frac{1}{2}B:\cos\frac{1}{2}C$$ 1+4=5

(b) Two forces P and Q acting at a point have got a resultant R; if Q be doubled, R is doubled. Again, if Q be reversed in direction, then also R is doubled. Show that

$$P:Q:R=\sqrt{2}:\sqrt{3}:\sqrt{2}$$ 5

(c) Three forces P, Q, R act in the same sense along the sides $\overrightarrow{BC}$, $\overrightarrow{CA}$, $\overrightarrow{AB}$ of a triangle ABC. Show that, if their resultant passes through the incentre, then $P+Q+R=0$. 5

2. (a) Show that the resultant of three equal like parallel forces acting at the angular points of a triangle passes through the centroid of the triangle. 5

(b) Show that the algebraic sum of the moments of the forces forming a couple about any point in their plane is a non-zero constant and equal to the moment of the couple. 5

(c) Define the moment of a force about a point. Show that the magnitude of the moment of a force about a point is represented by twice the area of the triangle formed by joining the point to the extremities of the line representing the force. 1+4=5

UNIT—II

3. (a) Forces of magnitudes 2, 4, 6, 8, $8\sqrt{2}$ act along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ and the diagonal $\overrightarrow{BD}$ of a square of side 2 units in the senses indicated by the order of the letters. Taking $\overrightarrow{AB}$, $\overrightarrow{AD}$ as the axes of x and y respectively, find the magnitude of the resultant force. 5

(b) A heavy rod is suspended from a point O by two strings OA and OB. Show that the plane OAB is vertical. 5

(c) How high can a particle rest inside a hollow sphere of radius a, if the coefficient of friction be $\frac{1}{\sqrt{3}}$? 5

4. (a) Find the centre of gravity of a uniform triangular lamina. 5

(b) $ABCD$ is a lamina in the form of a trapezium in which AB and CD are parallel and of lengths a and b respectively. Prove that the distance of the centre of gravity of $ABCD$ from the side AB is

$$\frac{h}{3}\cdot\frac{a+2b}{a+b}$$

h being the height of the trapezium. 6

(c) Define the following : 2×2=4

(i) Coefficient of friction

(ii) Angle of friction

UNIT—III

5. (a) At what distance from the centre will the velocity of a particle, executing simple harmonic motion, of amplitude a, be half of the maximum? 3

(b) A particle moves in a straight line. Its acceleration, directed towards a fixed point O in the line, is equal to $\mu\left(\frac{a^5}{x^2}\right)^{1/3}$, when it is at a distance x from O. If it starts from rest at a distance a from O, then show that it will arrive at O after a time $\frac{8}{15}\sqrt{\frac{6}{\mu}}$. 6

(c) Two smooth elastic spheres of masses m_1 and m_2 impinge directly when moving along the same line with velocities u_1 and u_2 respectively. Calculate the loss in kinetic energy due to the impact. 6

6. (a) It is known that earth attracts a body outside its surface with a force varying inversely as the square of the distance from the centre. A particle is attracted towards the centre of the earth, starting from rest of infinity. Find the velocity of the particle on reaching the earth's surface. 5

(b) A sphere impinges obliquely on another sphere at rest. If the two spheres are smooth, perfectly elastic and equal in mass, prove that they move at right angles to each other after impact. 5

(c) A particle of mass m is repelled with a force $m\mu x$, where x is its distance from the source of repulsion O. If the particle starts from rest at a distance a from O, where $x \geq a$; find its distance from O at time t. 5

UNIT—IV

7. (a) A particle is projected vertically upwards with velocity u in a medium whose resistance varies as the velocity. Find the greatest height attained by the particle. 5

(b) If at any point of the parabolic path of a projectile, the velocity be u and the inclination to the horizontal be θ, show that the particle is moving at right angles to its former direction after a time $\frac{u}{g}\operatorname{cosec}\theta$. 5

(c) A body is projected at an angle α to the horizon, so as just to clear two walls of equal height a at a distance $2a$ from each other. Show that the range is equal to $2a\cot\frac{\alpha}{2}$. 5

8. *(a)* The fluid resistance offered to the motion of a ship is given by the formula $R = av + bv^2$, where a and b are constants. The propulsion of a ship, weighing w, is stopped at the instant when $v = v_0$. Find the distance the ship will then move before coming to rest. 5

(b) A particle, of mass m, is projected vertically under gravity, the resistance of the air being mk times the velocity. Show that the time of ascent to the greatest height is $\frac{V}{g}\log(1+\lambda)$, where V is the terminal velocity of the particle and λV is its initial velocity. 5

(c) A particle is projected with velocity u at an angle α to the horizontal. Find the horizontal range of the particle. 5

UNIT—V

9. *(a)* Deduce the following work-energy equation, 'change in kinetic energy = work done by the forces', for a particle moving in a smooth plane curve under the action of conservative forces. 7

(b) A particle is projected from the lowest point of a smooth vertical circle of radius a with velocity u and moves along the inside of it. Apply the principle of energy to find the velocity of the particle at any angular distance θ from the lowest point. 4

(c) A heavy particle slides down a smooth cycloid (intrinsic equation $s = 4a\sin\psi$), starting from rest at a cusp, the axis being vertical and vertex downwards. Prove that the magnitude of the acceleration is equal to g at every point of the path. 4

10. *(a)* A point describes the cycloid $s = 4a \sin \psi$ with uniform speed u. Find its acceleration at any point in terms of u, a and s. 3

(b) A heavy particle of weight W, attached to a fixed point by a light inextensible string, describes a circle in a vertical plane. The tension in the string has values mW and nW respectively when the particle is at the highest and the lowest points of its path. Show that $n = m + 6$. 6

(c) A shot of mass m is fired from a gun of mass M with a velocity u relative to the gun. Show that the actual velocities of the shot and the gun are $\frac{Mu}{M+m}$ and $\frac{mu}{M+m}$ respectively and that their kinetic energies are inversely proportional to their masses. 6

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Answer **five** questions, taking **one** from each Unit

UNIT—I

1. (a) Find the magnitude and direction of the resultant of two forces P and Q acting at a point of a rigid body. 5

(b) The angle of inclination between two forces P and Q is θ. If P and Q be interchanged in position, show that the resultant will be turned through an angle ϕ, where

$$\tan\frac{\phi}{2} = \frac{P-Q}{P+Q}\tan\frac{\theta}{2}$$ 5

(c) Forces acting at a point are represented in magnitude and direction by $2AB$, 3*BC*, 2*CD*, *DA*, *CA* and *DB*, where *ABCD* is a quadrilateral. Show that the forces are in equilibrium. 5

2. (a) If the two like parallel forces *P* and *Q* $(P > Q)$ acting on a rigid body at *A* and *B* be interchanged in position, show that the point of application of the resultant be displaced along *AB* through a distance *d*, where

$$d = \frac{P-Q}{P+Q} \cdot AB$$

5

(b) Show that the moment of a force about a point is equal to the algebraic sum of the moments of its components about that point. 5

(c) Show that any number of coplanar couples acting on a body is equivalent to a single couple whose moment is equal to the algebraic sum of the moments of the couples. 5

UNIT—II

3. (a) Forces proportional to 1, 2, 3, 4 act along the sides *AB*, *BC*, *AD*, *DC* respectively of a square *ABCD*, the length of whose sides is 2 ft. Find the magnitude and the line of action of the resultant. 3+3=6

(b) A heavy uniform rod is in equilibrium with one end resting against a smooth vertical wall and the other against a smooth plane inclined to the wall at an angle θ. Prove that if α be the inclination of the rod to the horizon, then

$$\tan\alpha = \frac{1}{2}\tan\theta$$

5

(c) State the laws of statical friction. 4

4. (a) A uniform ladder rests with one end on the rough horizontal ground and the other against a rough vertical wall. The coefficient of friction at the lower and upper ends are $\frac{3}{7}$ and $\frac{1}{3}$ respectively.

Determine the angle which the ladder makes with the ground when it is about to slip. 6

(b) Define centre of gravity (c.g.). Prove that the c.g. of a body is unique. 1+2=3

(c) Find the c.g. of a uniform trapezium lamina. 6

UNIT—III

5. (a) A particle is projected vertically upwards from the earth's surface with a velocity just sufficient to carry it to infinity. Prove that the time it takes to reach a height h is

$$\frac{1}{3}\sqrt{\frac{2r}{g}}\left[\left(1+\frac{h}{r}\right)^{3/2}-1\right]$$

where r is the radius of the earth. 6

(b) A particle moves in a straight line, starting from rest at a distance a, towards the centre of force. If its acceleration at a distance x from the centre of force be $\mu / x^{5/3}$, show that it will reach the origin after a time

$$\frac{2a^{4/3}}{\sqrt{3\mu}}$$

5

(c) Two spheres of masses M, m impinge directly when moving in opposite directions with velocities u, v respectively. If the sphere of mass m is brought to rest by the collision, show that $v(m-eM)=M(1+e)u$. 4

6. (a) Two smooth imperfectly elastic spheres of masses m_1 and m_2 collide obliquely with velocities u_1 and u_2 making angles α_1 and α_2 with the line of centres. Calculate the loss in kinetic energy due to the impact. 6

(b) A particle rests in equilibrium under the attraction of two centres of force which attract directly as the distance, their intensities being μ, μ'. The particle is displaced slightly towards one of them. Show that the time of small oscillation is

$$\frac{2\pi}{\sqrt{\mu+\mu'}}$$

4

(c) In an SHM of period $\frac{2\pi}{\omega}$, if the particle be projected with a velocity u_0 from a point at a distance x_0 from the centre (away from the centre), prove that amplitude is

$$\left[x_0^2+\frac{u_0^2}{\omega^2}\right]^{1/2}$$

5

UNIT—IV

7. (a) A particle, of mass m, is falling under the influence of gravity through a medium whose resistance equals μ times the velocity. If the particle be released from rest, show that the distance fallen through in time t is

$$g\frac{m^2}{\mu^2}\left[e^{-\frac{\mu t}{m}}-1+\frac{\mu t}{m}\right]$$

5

(b) A particle of mass m moves from rest in a straight line under the action of a constant force in a medium whose resistance to the motion is $m(a+bv)$, where a and b are constants and v is the velocity at time t. If V be the terminal velocity, prove that the particle in time t has moved a distance x, where

$$bx = V(bt - 1 + e^{-bt})$$ 5

(c) A particle is projected from a point on the ground level and its height is h when it is at the horizontal distances a and $2a$ from its point of projection. Prove that the velocity of projection u is given by

$$u^2 = \frac{g}{4}\left[\frac{4a^2}{h} + 9h\right]$$ 5

8. (a) A particle is projected vertically upwards with a velocity u against a resistance proportional to the square of the velocity. If V is the terminal velocity of the body and m its mass, show that, when the body has fallen back to the point of projection, the loss of kinetic energy is

$$\frac{1}{2}mu^2\left(\frac{u^2}{V^2+u^2}\right)$$ 6

(b) A ball is projected so as to just clear two walls, the first of height a at a distance b from the point of projection and the second of height b at a distance a from the point of projection. Show that the range on the horizontal plane is

$$\frac{a^2+ab+b^2}{a+b}$$

and the angle of projection exceeds $\tan^{-1} 3$. 5

(c) If t be the time in which a projectile reaches a point P in its path and t' the time from P till it reaches the horizontal plane through the point of projection, show that the height of P above the horizontal plane is $\frac{1}{2}gtt'$. 4

UNIT—V

9. (a) A point P describes, with a constant angular velocity ω about O, the equiangular spiral $r = ae^{\theta}$, O being the pole of the spiral. Obtain the radial and transverse accelerations of P. 4

(b) A shell lying in a straight smooth horizontal tube suddenly breaks into two portions, of masses m_1 and m_2. If s

is the distance apart in the tube of the masses after time t, show that the work done by the explosion is

$$\frac{1}{2}\frac{m_1 m_2}{m_1+m_2}\cdot\frac{s^2}{t^2}$$ 5

(c) A particle is projected along the inner surface of a smooth vertical circle of radius a, its velocity at the lowest point being $\frac{1}{5}\sqrt{95ag}$. Show that it will leave the circle at an angular distance $\cos^{-1}\left(\frac{3}{5}\right)$ from the highest point and that its velocity then is $\frac{1}{5}\sqrt{15ag}$. 6

10. (a) A point moves along the arc of cycloid in such a manner that the tangent at its rotates with constant angular velocity. Show that the acceleration of the moving point is constant in magnitude. 4

(b) Show that all forces which are one-valued functions of distances from fixed points are conservative forces. 5

(c) Show that for a particle, sliding down the arc and starting from a cusp of a smooth cycloid whose axis is vertical and vertex lowest, the vertical velocity is maximum when it has described half the vertical height. 6

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